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Influence of Uncertainties in Structural Resistance on Glulam Girder Design

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Abstract

The objective of this technical paper is to show the influence of uncertainties in structural resistance and propose procedures to determine cross sections for glued-laminated timbers, glulam, based on reliability theory. Firstly the concept of limit coefficient of variation for structural resistance (Ω_R) was summarized. Below this limit, the structure could resist safely with the corresponding acceptable level of failure probability (p_f). Otherwise the expected structural reliability could not be achieved. Based on deflection limit states, the flexural rigidity (*EI*) could be constituted for the structural resistance. The uncertainty of *EI* could be statistically characterized by mean and coefficient of variation which is defined as the ratio of its standard deviation to its mean. Then concepts of transformed section associated with the elastic theory could be developed to determine glulam girders subjected to bending.

The applicability of the proposed concept could be demonstrated in numerical examples. It is not uncommon that the randomness of *EI* for composite glulam girders could be either normal or non-normal distribution depending on the statistical probability of species of wood combined. Then the fitted distribution of *EI* could be specified by Goodness-of-Fit Tests based on random experiments. If the lognormal distribution is fitted, the asymptotic Ω_R would be 0.155 for serviceability limit state ($p_f = 10^{-4}$). Unfortunately it is not uncommon that the coefficient of variation of the *EI* could exceed than this corresponding limit. As one of the most promising solution, the glulam composed of at least two species of wood may be used.

Keywords: glulam, uncertainty, structural resistance, limit of cov

1. Introduction

From series of tests at Rangsit University, e.g. [1], analyses showed that the modulus of elasticity of wood under tension was as high as 1.2-2.5 times that under compression. Furthermore, analyses of timber girders [2] indicated that the deflection limit states dominated the design of timber cross-sections. Therefore the flexural rigidity (*EI*) was the governing parameter for reliability analyses. The randomness of *EI* could be statistically characterized by mean and

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coefficient of variation which is defined as the ratio of its standard deviation to its mean.

Since statistical properties of *EI* for glulam girder reflected that of structural resistance, its limits of coefficient of variation of the structural resistance (Ω_R) depended on types of distribution as shown in [3]. For instance, the target failure probability of the serviceability limit state is 10^{-4}, the corresponding limit of Ω_R would be 0.155 and 0.255 for lognormal and gamma distribution, respectively. Below this limit, the central factor of safety (*FS*) for normal variate could be determined from the quadratic relationship between the coefficient of variation for structural resistance and load effects. Since wood is naturally occurring material, mechanical properties and its variations could not be controlled. Unfortunately it is not uncommon that the coefficient of variation of *EI* could exceed than this corresponding limit. In the case of low *EI* and/or high coefficient of variation, beams may not be used safely against pre-specified target failure probabilities. In this case, the glulam girder composed of at least two species of wood may be used.

It is quite common that *EI* of composite glulam girder could be non-normal distribution depending on the statistical probability of species of wood combined. Generally the type of distribution and the dispersion of structural resistance could not be obtained directly from simple statistical analyses. Fortunately, the above mentioned statistical properties could be obtained easily from random experiments, i.e. Monte Carlo Simulation Technique. Then most suitable distribution could be specified by Goodness-of-Fit Tests with existing software, e.g. CESTEST [4].

Based on assumptions of perfect bonding among horizontal layers and vertical joints as well as cross-section remaining plane during bending, concepts of transformed section could be applied to determine the appearance *EI* of composite glulam girders.

2. Basic Concept of Structural Reliability

For time invariant reliability analyses failure probability (p_f) may be defined as the probability that the structural resistance *(R)* will not be exceeded by load effects *(S)* within the whole service life as shown:

$$p_f = Pr\,(R\text{-}S > 0) \tag{1}$$

In this case the limit state function of a structural system $g(\underline{X})$ may be defined as:

$$g(\underline{X}) = R - S \tag{2}$$

Where $\underline{X}$ is a vector of random variable. $g(\underline{x}) > 0$ defines safe state and defines failure state, otherwise. For further details it is referred to [5] and [6].

3. Failure Probability and Safety Index

Let R and S be uncorrelated normal variables. Their randomness could be characterized by mean μ_R and μ_S, and corresponding standard deviation σ_R and σ_S, respectively. Its mean value, μ_g, and standard deviation, σ_g, can be expressed as in Eq. 3 and Eq. 4, respectively.

$$\mu_g = \mu_R - \mu_S \tag{3}$$

$$\sigma_g = \sqrt{(\sigma_R)^2 + (\sigma_S)^2} \tag{4}$$

For the limit state, where $g(\underline{x}) = 0$, the safety index or the reliability index (β) could be obtained from the ratio of its mean value to standard deviation as shown in Eq. 5. The higher value of β implies lower value of failure probability as shown in Fig. 1.

$$\beta = \frac{\mu_g}{\sigma_g} \tag{5}$$

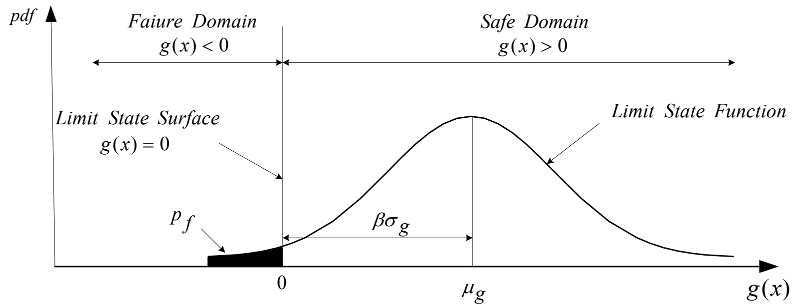


Fig. 1 Relationship between Safety Index and Failure Probability

4. Limit of Ω_R for Normal Variate

Let *R* and *S* be normally distributed. By substituting μ_g and σ_g from Eq. 3 and Eq. 4 into Eq. 5 the safety index may be rewritten as in Eq. 6.

$$\beta = \frac{\mu_R - \mu_S}{\sqrt{(\sigma_R)^2 + (\sigma_S)^2}} \tag{6}$$

A standard measure for the dispersion about mean is the coefficient of variation (Ω), which is defined as the ratio of its standard deviation to its mean. Defining $\Omega_R = \sigma_R / \mu_R$, $\Omega_S = \sigma_S / \mu_S$ and the central factor of safety, $FS = \mu_R / \mu_S$, Eq. 6 could be rearranged in terms of the central factor of safety and the coefficient of variation as:

$$FS - 1 = \beta\sqrt{(FS)^2(\Omega_R)^2 + (\Omega_S)^2} \tag{7}$$

The relationship between *FS* and Ω_R for given values of Ω_S can be shown in Fig. 2.

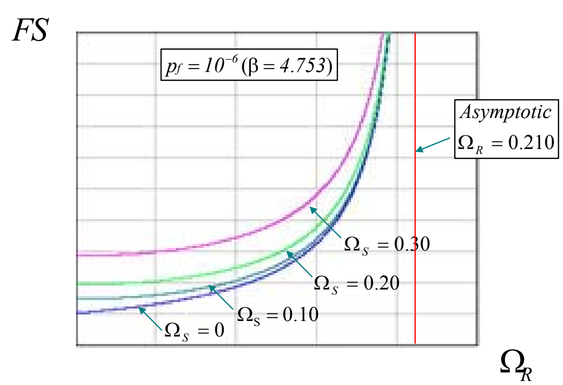


Fig. 2 Relationship between Ω_R, Ω_S and *FS* for $p_f = 10^{-6}$

Fig. 2 shows that the asymptote for Ω_R is independent of variation of load effects. If the Ω_R is higher than the limit of Ω_R, the structural system might not be safe for the expected level of risk. It is interesting to note that for the case $\Omega_S = 0$ and lower value of Ω_R, the structural system could be used safely for a pre-specified target reliability with lower bound of *FS*.

5. Limit of Ω_R for Non-normal Structural Resistance

If the structural resistance is not normally distributed, the limit of Ω_R may be obtained from the equivalent normal distribution concepts. The limit values of Ω_R for other types of commonly used distribution for structural response are summarized in Table 1.

Table 1 Limit of Ω_R for Commonly Used Types of Distribution for Structural Resistance

Distribution	Ω_R					
	$p_f = 10^{-2}$	$p_f = 10^{-3}$	$p_f = 10^{-4}$	$p_f = 10^{-6}$	$p_f = 10^{-8}$	$p_f = 10^{-10}$
Lognormal	0.253	0.187	0.155	0.120	0.102	0.090
Gamma	0.370	0.301	0.255	0.204	0.174	0.154
Normal	0.430	0.324	0.269	0.210	0.178	0.157
Weibull	0.402	0.350	0.324	0.295	0.280	0.269

Topics are discussed in details by author in [7] for the context of limit of the structural resistance and its applications.

6. Transformed Section of GluLam Girder

Since the modulus of elasticity of wood under tension (E_t) is relatively higher than compression (E_c), a timber beam subjected to bending could not be determined as a homogenous beam of one material. For the horizontal layer and vertical joint are perfectly bonded, the assumptions of elastic theory for homogeneous beam are valid to analyze glulam girder. In case of glulam girders the original cross-section could be transformed to an equivalent cross sections for one homogeneous material in term of modular ratio, $n=E_t/E_c$, as shown in Fig. 3.

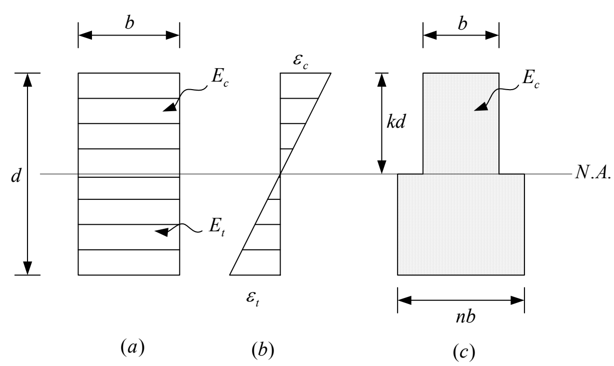


Fig. 3 Glulam Girder: (a) Cross-Section; (b) Transformed Section.

The location of *kd* could be obtained from a summation of the first moment of the transformed sections with respect to the neutral axis is zero. Thus that,

$$(n-1)k^2 - 2nk + n = 0 \tag{8}$$

The moment of inertia for compression *(I_c)* and tension portion *(I_t)* about neutral axis can be obtained from:

$$I_c = \frac{b(kd)^3}{12} + \frac{b(kd)^3}{4} \tag{9}$$

$$I_t = \frac{b(d-kd)^3}{12} + \frac{b(d-kd)^3}{4} \tag{10}$$

The moment of inertia about the neutral axis of the transformed sectional may be defined as:

$$I = I_c + nI_t \tag{11}$$

Introducing I_c, I_t and $n=E_t/E_c$, the flexural rigidity (EI) for equivalently homogeneous girder can be further rewritten as in Eq. 12 and Eq. 13.

$$EI = E_c I_c + E_t I_t \tag{12}$$

$$EI = E_c(I_c + nI_t) \tag{13}$$

$$EI = \frac{1}{3} bE_c \left[(kd)^3 + \frac{E_t}{E_c}(d-kd)^3 \right] \tag{14}$$

Where a higher structural resistance is required, i.e. beam subjected to high load effects and long span beam, the increment of *E*, instead of *I*, may be a feasible alternative to improve *EI*. For this purpose, a common practice, a lower strength timber would be faced with higher grade at outer laminations. In reliability senses a variation of glulam girder could be reduced by lamination with other woods of lower variation. Therefore higher grades of outer laminations are not only aimed for resisting flexural stresses but also proposed to limit the variation of Ω_R. The new cross-section of a homogeneous material with the modulus of elasticity E_c could be constructed as shown in Fig. 4.

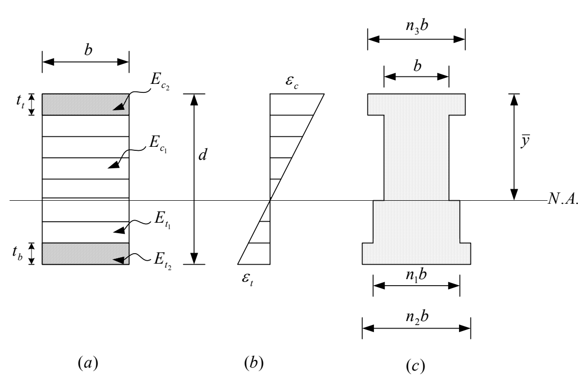


Fig. 4 Sandwiched Girder: (a) Typical Cross-Section; (b) Transformed Section.

Where $n_1=E_{t1}/E_{c1}$, $n_2=E_{t2}/E_{c1}$ and $n_3=E_{c2}/E_{c1}$. The centroid of a transformed section, $\bar{y}$, must be satisfied by the condition of the first moment of transformed sections. The moment of inertia of compression and tension portion about neutral axis can be obtained from:

$$I_{c1} = \frac{1}{12}(b)(\bar{y} - t_t)^3 + (b)(\bar{y} - t_t)(\frac{\bar{y} - t_t}{2})^2 \tag{14}$$

$$I_{c2} = \frac{1}{12}(b)(t_t)^3 + (b)(t_t)(\bar{y} - \frac{t_t}{2})^2 \tag{15}$$

$$I_{t1} = \frac{1}{12}(b)(d - \bar{y} - t_b)^3 + (b)(d - \bar{y} - t_b)(\frac{d - \bar{y} - t_b}{2})^2 \tag{16}$$

$$I_{t2} = \frac{1}{12}(b)(t_b)^3 + (b)(t_b)(d - \bar{y} - \frac{t_b}{2})^2 \tag{17}$$

The flexural rigidity for homogeneous girder from Eq. 12 can be rewritten as:

$$EI = E_{c1}\left(I_{c1} + n_3 I_{c2} + n_1 I_{t1} + n_2 I_{t2}\right) \tag{18}$$

7. Limit State Function of Timber Girders

Analyses of long span beams show that the limit of deflection plays more major role for designing cross-section than the allowable stress. Therefore the serviceability limit state could be considered for reliability analyses of a glulam girder. The limit for an elastic deflection at the midspan of a simply supported girder subjected to uniformly distributed loads can be calculated from Eq. 19.

$$\Delta = \frac{5wL^4}{384EI} \tag{19}$$

Since the limit of elastically vertical deflection at midspan is considered for reliability analyses, the limit state function can be written as:

$$\begin{aligned} g(x) &= \frac{L}{360} - \frac{5wL^4}{384EI} \\ g(x) &= EI - \frac{75}{16}wL^3 \end{aligned} \tag{20}$$

Eq. 20 implies that the structural resistance of beam corresponding to the limit of elastically vertical deflection could be represented by the flexural rigidity.

8. Monte Carlo Simulation

If the limit state function is complicate, the corresponding response could not be obtained by direct integration. Therefore the Monte Carlo simulation technique is proposed to obtain the mean value of limit state function. In this method, the cumulative distribution function [$F_X(x_i)$] of any continuous variate is definite to the uniform random numbers (u_i) over the interval [0, 1]. Thus that,

$$u_i = F_X(x_i) \tag{21}$$

Then inverse transformations of the cumulative distribution function [$F_X^{-1}(x_i)$] is employed for solving random numbers (x_i). It can be expressed that

$$x_i = F_X^{-1}(x_i) \tag{22}$$

For instance, if E is Weibull distribution the transformation for Weibull random number could be interpreted as:

$$u_i = F_X(E_i) = 1 - e^{-(E_i/\nu)^m}$$
$$E_i = \nu\left[-\ln u_i\right]^{1/m} \tag{23}$$

Where υ and m are the parameter of Weibull distribution. Thus, E_i corresponding to the Weibull distribution can be generated using Eq. 23.

9. Numerical Examples

Example 1. A simply supported glulam girder, L=5 m, subjected to uniform load intensity of 300 kg/m as shown in Fig. 5. The girder made by bonding together multiple Pradipat planks (***Casuarina Junghuhniana*** Miq.) of 2 cm × 10 cm × 30 cm. The density (ρ) of Pradipat planks is 0.75 g/cm^3. The E_t and E_c of Pradipat wood could be characterized by Weibull and gamma distribution with mean values estimated to be 1.33×10^5 ksc and 9.65×10^4 ksc, respectively. The corresponding coefficients of variation are 0.18 and 0.19 respectively. For the sake of simplicity, variables L, b, d and ρ are assumed to be deterministic. The appropriated section of girder is needed to assure that the probability of midspan deflections will not exceed serviceability limit state at acceptable failure probability of 10^{-4}.

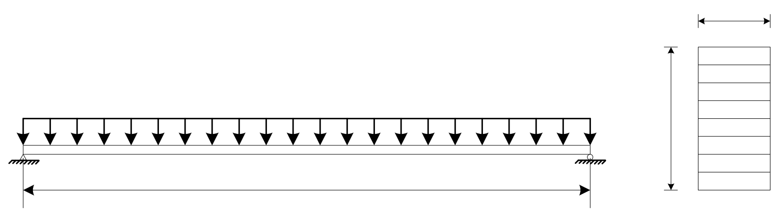


Fig. 5 Glulam Girder Subjected to Distributed Load

The total uniform load, in *kg/cm*, could be estimated as:

$$w = 3 + \frac{\rho b d}{1000} \tag{24}$$

Based on uncertainties of E_t and E_c the randomness of k may be characterized by lognormal distribution, using the Monte Carlo Simulation Technique, through Eq. 8 with mean value of 0.541 and the corresponding coefficient of variation value of 0.03. Since k is less sensitive, coefficient of variation < 0.1, it can assume to be constant. Substituting EI from Eq. 13 into Eq. 20, the limit state function can be rewritten in the following form:

$$g(x) = \frac{L}{360} - \frac{5wL^4}{384\frac{b}{3}\left[E_c(kd)^3 + E_t(d-kd)^3\right]}$$

$$g(x) = \frac{b}{3}[E_c(kd)^3 + E_t(d-kd)^3] - \frac{75}{16}(3 + \rho bd)L^3 \tag{25}$$

Then the structural resistance could be represented as:

$$R = \frac{b}{3}[E_c(kd)^3 + E_t(d-kd)^3] \tag{26}$$

Where L is the span length (500 cm), w is the total uniform load, ρ is the density of Pradipat plank (0.75 g/cm^3), and b is the width of girder (10 cm). Numbers of failure (N_f) may be obtained using the Monte Carlo Simulation Technique through Eq. 25. For simulation numbers (N) of 2^{15} the probability of failure, N_f/N, corresponding to the depth of girder can be shown in Fig. 6.

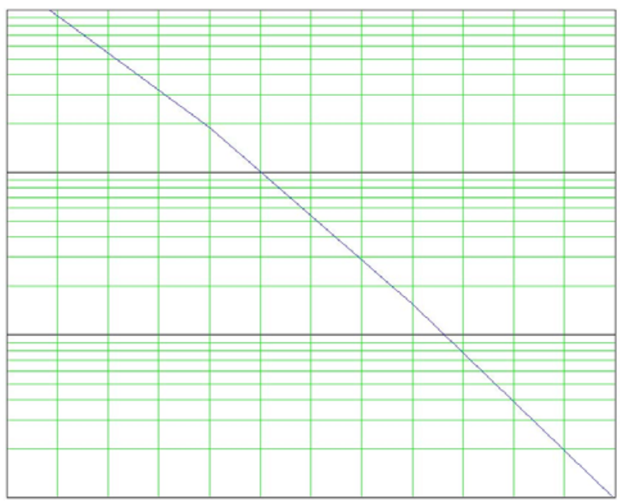


Fig. 6 Probabilities of Failure Corresponding to Depths of Girder

Fig.6 shows that the probability of failure decreases with the increment of girder depth. However, only lower values of d corresponding to the target failure probability, less than 1×10^{-4}, are interested. If d = 36 cm is selected, the corresponding failure probability is found to be 1.62×10^{-5} ($\beta = 4.156$).

On the other hand the structural reliability of this glulam girder could be examined by means of the limit of Ω_R. The randomness of structural resistance could be characterized by the Monte Carlo Simulation Technique through Eq. 26. Based on statistical data and 1024 (2^{10}) simulations, the fitted distribution of outcomes could be obtained from CESTTEST Software. For a confidence interval of 99% the normal distribution could be accepted by Chi-Square and K-S Test as shown in Fig. 7.

10^{-3}

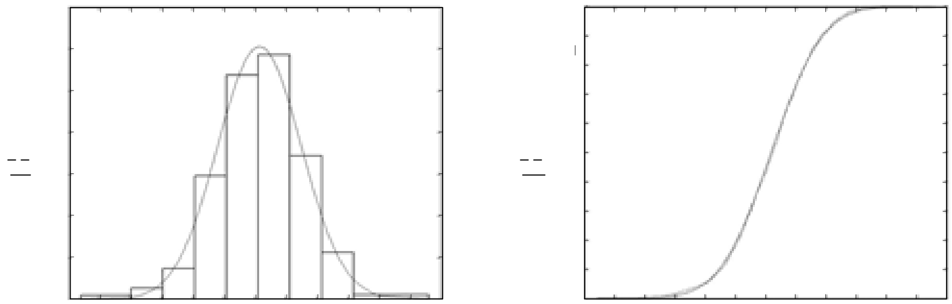


Fig. 7 Goodness of Fit Test for Normal Distribution: (a) Chi-Square, (b) K-S Test.

The randomness of the structural resistance could be characterized by $\mu_R = 4.6\times10^9$ kg.cm^2 and $\Omega_R = 0.13$. For the fitted distribution of structural resistance is normal the limit of Ω_R is 0.269 (p_f = 10^{-4}). Substituting β = 4.156, $\Omega_R = 0.13$ and Ω_S = 0 in to Eq. 7, *FS* is found to be 2.18. Since Ω_R of the structural system is less than the limit, the glulam girder with 36 cm in depth made from Pradipat planks could be used safety against serviceability limit state with *FS* = 2.18.

Example 2. Laminated Durian planks are proposed for girder of 5.5 m span as shown in Fig. 8. Both E_t and E_c could be characterized by the lognormal distribution with mean values of 1.26×10^5 ksc and 8.12×10^4 ksc, respectively. The corresponding coefficients of variation are 0.17 and 0.19, respectively.

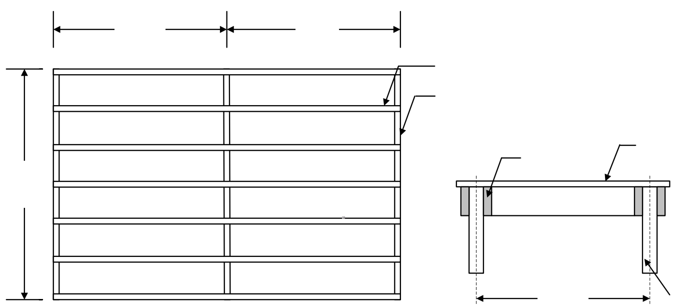


Fig. 8 Fixed End Girder Subjected to Distributed Load

The density of Durian plank (ρ_D) is estimated to be 0.51 g/cm^3. The girder is subjected to dead loads and live loads totally 4 kg/cm excluding its self weight. The width of girder is limited to 10 cm and the design depth of girder, including deflections, should not exceed limit value of 40 cm due to reserving space for utilities above the ceiling. The outer lamination is needed to improve the structural resistance and suit the design criteria.

The elastic deflection at midspan of a fixed end girder subjected to uniformly distributed loads can be calculated from Eq. 27.

$$\Delta = \frac{wL^4}{384EI} \tag{27}$$

The limit state function can be further written as:

$$g(x) = EI - \frac{15}{16}wL^3 \tag{28}$$

$$g(x) = \frac{b}{3}[E_c(kd)^3 + E_t(d-kd)^3] - \frac{15}{16}(4 + 0.51\frac{bd}{1000})(550)^3 \tag{29}$$

Employing a procedure similar to that used in the preceding numerical example, it is possible to develop expressions in Eq. 29 relating the Monte Carlo Simulation Technique to the failure probability. Based on statistical data and simulation numbers of 2^{15} the probability of failure corresponding to depths of girder can be shown in Table 2.

Table 2 Summary of Simulation Results

Girder Section $b \times d$ (cm×cm)	Probability of Failure (p_f)	Δ (cm)	Depth+ Δ (cm)
10×36	1.13×10^{-1}	1.14	37.14
10×38	8.09×10^{-3}	0.97	38.97
10×40	6.10×10^{-5}	0.83	40.83

Observe from Table 2 the given cross-sections could not meet the design criteria. Since the width and depth of girder are limited, the increment of E is an alternative to improve the structural stiffness. The girder may be sandwiched with a hard wood of higher strength separated by a relatively thick layer of Durian plank. This girder may be faced with Para or Rubber wood as shown in Fig. 9, for instance.

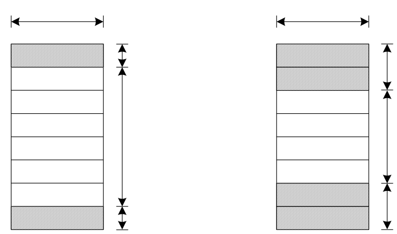


Fig. 9 Laminations of Durian and Para planks: (a) Single Layer, (b) Double Layers

The randomness of E_t and E_c could be characterized by Weibull and gamma distribution with mean values of 1.46×10^5 ksc and 9.5×10^4 ksc. The corresponding coefficients of variation are 0.17 and 0.21, respectively. The Para plank (ρ_P) density is estimated to be 0.54 g/cm^3.
The total uniform load, in *kg/cm*, could be estimated from

$$w = 4 + \frac{\rho_D bd}{1000} + \frac{\rho_P b(t_t + t_b)}{1000} \tag{30}$$

Substituting EI from Eq. 18 into Eq. 28, the limit state function can be written in the following form.

$$g(x) = E_c \left(I_{c1} + I_{c1} + I_{t1} + I_{t2} \right) - \frac{15wL^3}{16} \qquad (31)$$

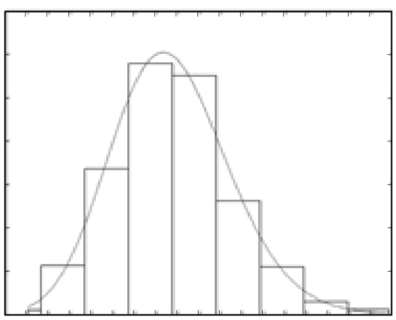
Substituting expressions of Eq. 14, Eq. 15, Eq. 16 and Eq. 17 in Eq. 31, then the limit state function is ready for performing failure probability. Based on statistical data and simulation numbers of 2^{15} the failure probability corresponding to combined depth can be shown in Table 3.

Table 3 Summary of Simulation Results

d (cm)	Para Plank		Durian Plank	Failure Probability (p_f)
	t_t (cm)	t_b (cm)	d- t_t - t_b (cm)	
36	2	2	32	2.37×10^{-4}
38	2	2	34	7.63×10^{-6}
36	4	4	28	1.10×10^{-2}
38	4	4	30	9.92×10^{-5}
40	4	4	32	1.03×10^{-7}

Table 3 shows that failure probability of girder using double layers of outer laminations is lower than single layer of lamination at the same gross depth. However, the determination of use depended on the acceptable level of risk and cost-benefit aspects.

For double layers of combination depth of 38 cm the randomness of *EI* could be obtained from Eq. 18. The number of simulations is 2^{10}. For a confidence interval of 99% the lognormal distribution could be accepted as shown in Fig.10.



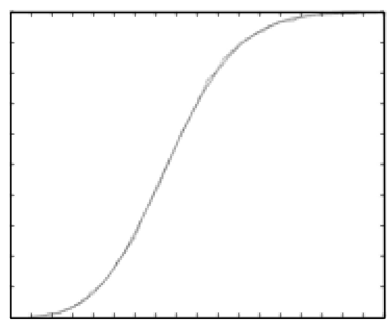


Fig. 10 Goodness of Fit Test for Lognormal: (a) Chi-Square, (b) K-S Test.

The randomness of the structural resistance could be characterized by $\mu_R = 3.4\times10^9$ kg.cm^2 and $\Omega_R = 0.12$. Since the structural resistance is non-normal, Eq. 7 is not valid for obtaining the central factor of safety. In this case the direct integration approach may be applied for obtaining value of μ_S. The concept of finding μ_S corresponding to $p_f = 6.1\times10^{-5}$ could be interpreted in Fig. 11.

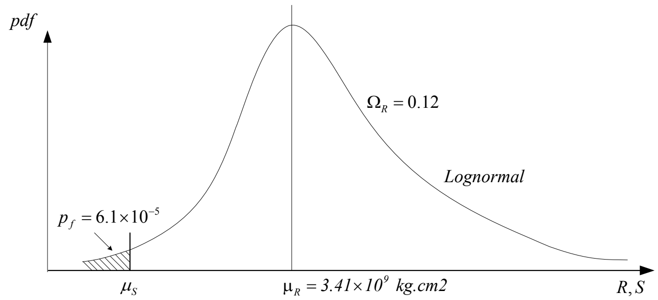


Fig. 11 Concepts of Integration for Finding μ_S

The μ_S is found to be 2.1×10^9 kg.cm^2 and FS becomes 1.62. When the fitted distribution of structural resistance is lognormal the limit of Ω_R using the concept of equivalent normal distribution become 0.155 (p_f=10^{-4}). Since Ω_R of the structural system is less than the limit this glulam girder could be used safety against serviceability limit state with FS = 1.62.

For the lamination with hard wood of low variation, it is clear that the coefficient of variation of structural resistance is decreased. The assumption of this reduction technique is widely applied for other composite structures, e.g. timber beam with steel wire or plastic band and reinforced concrete.

10. Conclusion

1. The classical reliability analyses in terms of normally structural resistance and load effects show that there should be an asymptotic coefficient of variation of structural resistance (Ω_R) for the corresponding target failure probability (p_f).
2. If coefficient of variation of the structural resistance is below this limit, the structure could resist safely with the corresponding acceptable level of risk. The central safety factor for a particular value of coefficient of variation of load effects could be determined from the quadratic relationship. For non-normal structural resistance the central factor of safety may be obtained using the direct integration concepts.
3. For the modulus of elasticity of wood under tension (E_t) is relatively higher than that under compression (E_c), the timber beam subjected to bending could not be determined as a homogenous beam of one material.
4. The concepts of transformed section for a composite beam could be used to determine a timber having different modulus of elasticity. Since the horizontal layer and vertical joint are perfectly bonded, the assumptions of elastic theory for homogeneous beam are valid to analyze a glued-laminated girder (glulam).
5. For the serviceability limit state, the flexural rigidity (EI) represents the structural resistance of the glued-laminated girder subjected to bending.
6. The randomness of structural resistance for glued-laminated girder could be obtained by Monte Carlo Simulation Technique. Then the fitted distribution of structural responses could be confirmed by the Goodness-of-Fit Tests, i.e. Chi-Square Test and K-S Test. For non-normal structural resistance, fitted distributions could be obtained easily with CESTTEST Software.
7. Since wood is a naturally occurring material, variations of its mechanical properties could not be controlled. In reliability senses a variation of structural system could be reduced by lamination with other woods of lower variation.

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