International Journal of
Materials
& Structural Reliability

International Journal of Materials & Structural Reliability Vol.3, No.2, September 2005, 87 - 94

Crack Width Due to Corroded Bar in Reinforced Concrete Structures

C.Q. Li[1*], W. Lawanwisut[2], J.J. Zheng[3] and W. Kijawatworawet[4]

[1*] *Division of Civil Engineering, University of Dundee, Dundee, DD1 4HN, UK*
[2] *IMMS Co., Ltd., Rachadapisek Rd., Klongtuey, Bangkok 10110 Thailand*
[3] *Faculty of Civil Engineering, Zhejiang University of Tech., P R China*
[4] *Faculty of Civil Engineering, Kasetsat University, Nakhon Pathom, 73140 Thailand*

Abstract

Practical experience and observations suggest that corrosion affected reinforced concrete structures are more prone to cracking than other forms of structural deterioration. Determination of corrosion induced crack width is essential to the prediction of the serviceability of corrosion affected concrete structures and to the decision-making regarding the repairs for the structures. Although considerable research has been undertaken on corrosion induced cracking process, little work has been carried out directly on corrosion induced crack width both numerically and experimentally, and no analytical model for the crack width has been published to date. This paper attempts to investigate the corrosion induced cracking process in an analytical manner. In this paper, a theoretical model for corrosion induced crack width in reinforced concrete structures is derived. Fracture mechanics is employed for the analysis of stress and strain in the concrete surrounding the reinforcing bar. A merit of the derived model is that it is directly related to critical factors that affect the corrosion induced cracking process. The derived model is verified with both experimental and numerical data obtained from research literature. The model derived in the paper can serve as a useful tool for engineers, operators and asset managers in decision-making regarding the maintenance and repairs of corrosion affected reinforced concrete infrastructure.

Keywords: Concrete structure, Cracking, Reinforcement corrosion, Serviceability

1. Introduction

Corrosion of reinforcing steel in concrete is the predominant causal factor in the premature degradation of reinforced concrete (RC) structures (Broomfield, 1997). Practical experience and observations (Liu and Weyers, 1997 : Li, 2003) suggest that, although many RC structures are seen as "badly" deteriorated, characterized by mass concrete cracking and spalling, they are still structurally sound. The reason for this is attributed to the nature of the problem; the corrosion products exert an expansive stress on concrete the tensile strength of which is usually low. It is also partially due to the fact that the safety factors used in structural design for strength are usually larger than those for serviceability since the paramount importance of structural safety. As a result, corrosion affected RC structures are more prone to cracking (than, e.g., loss of strength), incurring considerable costs of repairs and inconvenience to the public due to interruptions. This gives rise to the need for thorough

* Corresponding author.
E-mail : c.q.li@dundee.ac.uk

investigation on corrosion induced cracking process in order to achieve cost-effectiveness in maintaining the serviceability of the RC structures.

Considerable research has been undertaken on corrosion induced cracking process, with perhaps more numerical and experimental investigations than analytical ones. Numerical investigations use mainly finite element methods with various models for the growth of corrosion products (i.e., expansive pressure) and concrete behavior once cracked. For example, Dagher and Kulendran (1992) and Pantazopoulou and Papoulia (2001) assume that the cracks in concrete are smeared and employ fracture mechanics to determine the stress in concrete and hence the cracking. Molina et al. (1993) model the concrete as linear softening material and the corrosion products as a layer with reduced modulus of elasticity. With this model, the length of the crack evolution in concrete can be determined. Noghabai (1996) and Coronelli (2002) also combine the corrosion induced pressure (stress) with the pressure produced by bond action in determining the stress in concrete. Ueda et al. (1998) use finite element method to examine the factors that affect corrosion induced cracking in concrete and find that the tensile strength and creep of concrete are important factors.

In experimental investigations on corrosion induced cracking in concrete, the corrosion process is usually accelerated by various means so that concrete cracking can be achieved in a relatively short time (Alonso et al. 1998 : Andrade et al. 1993 : Liu and Weyers, 1998 : Francois and Arliguie, 1998). Most of the experiments appear to focus on surface cracking of concrete rather than on direct measurement of crack width over time (Liu and Weyers, 1998). Once experimental results are produced, empirical models can be readily developed for corrosion induced concrete cracking based on mathematical regression, including both deterministic models, such as Alonso et al. (1998), Rodriguez et al. (1996), and probabilistic models, such as Thoft-Christensen (2001) and Vu and Stewart (2002). The factors that affect the corrosion induced cracking have also been studied in experiments. For example, Alonso et al. (1998) find that, in addition to concrete properties, the corrosion rate and cover to bar diameter ratio are critical factors.

As is well known, crack width is a parameter of the most practical significance for the design and assessment of RC structures, Therefore, the determination of corrosion induced crack width is essential to the prediction of the serviceability of corrosion affected RC structures and to the decision-making regarding the repairs for the structures. Although considerable research has been undertaken on corrosion induced cracking process, a study of research literature (see about references) suggests that little work has been carried out directly on corrosion induced crack width both numerically and experimentally, and no analytical model for the crack width has been published to date. While acknowledging that empirical models exist, e.g., Alonso et al. (1998), for widespread application of cracking models to corrosion affected RC structures by a variety of users, it is more desirable to have analytical models (e.g., Bažant, 1979 : Liu and Weyers, 1998).

Moreover, for corrosion affected RC structures, repairing costs due to concrete cracking and spalling exceed those from other forms of deterioration by a substantial margin (Dhir and McCarthy, 1999). It is therefore imperative to accurately predict the crack width in order to achieve the cost effectiveness in the asset management of RC structures. It is in this regard that the present paper attempts to investigate the corrosion induced cracking process in an analytical manner, in which a theoretical model for corrosion induced crack width in RC structures is derived. In this paper, fracture mechanics is employed for the analysis of stress and strain in the concrete surrounding the reinforcing bar. Corrosion induced cracks in concrete are assumed to be smeared and the concrete is considered to be quasi-brittle material. A merit of the derived model is that it is directly related to critical factors that affect the corrosion induced cracking process, such as the corrosion rate, concrete geometry and property. The derived model is also verified with both experimental and numerical data obtained from research literature.

2. Corrosion Induced Concrete Cracking

As is well known, concrete with embedded reinforcing steel bars can be modeled as a thick-wall cylinder (Bažant, 1979 : Tepfers, 1979). This is shown schematically in Fig. 1(a), where D is the diameter of reinforcement bar, d_0 is thickness of the annular layer of concrete pores (i.e., a pore band) at the interface between the reinforcing bar and concrete, and C is the concrete cover. Usually d_0 is constant once concrete has hardened. The inner and outer radii of the thick-wall cylinder are $a =$

$(D+2d_0)/2$ and $b = C + (D+2d_0)/2$. When the reinforcing steel corrodes in concrete, its products (i.e., rusts, mainly ferrous and ferric hydroxides) fill the pore band completely. As the corrosion propagates in concrete, a ring of corrosion products forms, the thickness of which, $d_s(t)$ (Fig. 1(b)), can be determined from (Liu and Weyers, 1998)

$$d_s(t) = \frac{W_r(t)}{\pi(D+2d_0)}\left(\frac{1}{\rho_r} - \frac{\alpha_r}{\rho_{st}}\right) \tag{1}$$

where α_r is a coefficient related to the type of corrosion products, ρ_r is the density of corrosion products, ρ_{st} is the density of the steel and $W_r(t)$ is the mass of corrosion products. Obviously, $W_r(t)$ increases with time and can be determined from (Liu and Weyers, 1998)

$$W_r(t) = \left[2\int_0^t 0.105\left(\frac{1}{\alpha_r}\right)\pi D i_{corr}(t)dt\right]^{1/2} \tag{2}$$

where $i_{corr}(t)$ is the corrosion current density (in μA/cm^2) which is a measure of corrosion rate.

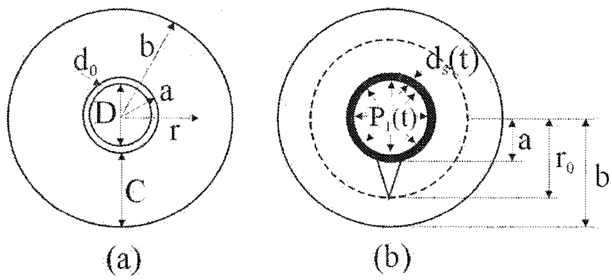

Fig. 1 Schematic representation of cracking process

The growth of the ring of corrosion products (known as rust band) exerts an outward pressure on the concrete at the interface between the rust band and concrete. Under this expansive pressure, the concrete cylinder undergoes three phases in terms of cracking: (i) no cracking; (ii) partially cracked; and (iii) completely cracked. In the phase of no cracking, the concrete cylinder can be considered to be elastic isotropic so that the theory of elasticity can be used to determine the radial stress $\sigma_r(r)$ and tangential stress $\sigma_\theta(r)$ at any point (r) in the cylinder (Timoshenko and Goodier, 1970). From the radial stress $\sigma_r(r)$, the expansive pressure at the interface between the rust band and concrete can be obtained as

$$P_1 = -\sigma_r(a) = \frac{E_{ef} d_s(t)}{a\left(\frac{b^2+a^2}{b^2-a^2} + \upsilon_c\right)} \qquad 3)$$

where E_{ef} is the effective elastic modulus of concrete and υ_c is Poisson's ratio of concrete. It may be noted that Equation (3) is the same as that was reported in Liu and Weyers (1998). From the tangential stress $\sigma_\theta(r)$ at $r = a$, the initial cracking time can be determined by satisfying the condition $\sigma_\theta(a) = f_t$, where f_t is the tensile strength of concrete.

After cracking initiation, the crack in the concrete cylinder propagates along a radial direction and stops arbitrarily at r_0 (which varies between the radii a and b) to reach a state of self-equilibrium. The crack divides the thick-wall cylinder into 2 co-axial cylinders: inner cracked and outer uncracked ones, as shown in Fig. 1(b). For the outer uncracked concrete cylinder, the theory of elasticity still applies. For the inner cracked concrete cylinder, let it now be assumed that the cracks are smeared and uniformly distributed circumferentially in the cracked cylinder (Pantazopoulou and Papoulia, 2001). Also let it be assumed that the concrete is a quasi-brittle material. With these assumptions, fracture mechanics can be applied to determine the stress distribution in the cracked cylinder (Kanninen and Popelar, 1985). According to Bažant and Jirasek (2002) and Noll (1972), there exists a residual tangential stiffness in the cracked concrete. Since the residual tangential stiffness at each point on the cracked surface along the radial direction is dependent on the tangential strain of that point, it is a function of the radial co-ordinate r. In view of the lack of knowledge of the residual stiffness of the cracked concrete, it is assumed in this paper that the residual tangential stiffness is constant along the cracked surface, i.e., on the interval (a, r_0), and represented by αE_{ef}, where α (<1) is tangential stiffness reduction factor. Based on Bažant and Planas (1998), the stiffness reduction factor α is dependent on the average tangential strain $\overline{\varepsilon_\theta}$ over the cracked surface and can be determined as follows (also see Sheng et al. 1991)

$$\alpha = \frac{f_t \exp\left[-\gamma\left(\overline{\varepsilon_\theta} - \overline{\varepsilon_\theta^c}\right)\right]}{E_{ef}\overline{\varepsilon_\theta}} \tag{4}$$

where $\overline{\varepsilon_\theta^c}$ denotes the average tangential cracking strain and γ is a material constant. The cracking in radial direction makes the concrete an anisotropic material locally in the vicinity of cracks. That is, the elastic modulus in the radial direction is different from that in the tangential direction. Li et al. (2004) developed a formula for crack width based on the concept of fracture mechanic and the well-known model of thick-wall cylinder as shown in Fig. 1. According to Li et al. (2004). The corrosion induced concrete crack width (w_c) can be expressed as follows

$$w_c = \frac{4\pi d_s(t)}{(1-\upsilon_c)\left(\dfrac{a}{b}\right)^{\sqrt{\alpha}} + (1+\upsilon_c)\left(\dfrac{b}{a}\right)^{\sqrt{\alpha}}} - \frac{2\pi b f_t}{E_{ef}} \tag{5}$$

In Equation (5), the key variables are the thickness of corrosion products d_s and the stiffness reduction factor α. d_s is directly related to the corrosion rate as shown in Equations (1) and (2). α is related to concrete geometry and property. Obviously, with the accumulation of corrosion products, the crack width increases. This makes sense both theoretically as shown in Equation (5) and practically as experienced and observed (Andrade et al. 1993 : Liu and Weyers, 1998). It needs to be noted that, due to the random nature of crack occurrence, there may be more than one crack occurring either simultaneously or within a short period of time. In this case, the assumption that the crack width of all cracks is equal could be made according to Molina et al. (1993). Thus Equation (5) is still applicable but w_c should be divided equally by the number of cracks. In any event, Equation (5) represents the maximum crack width on the surface of concrete. Using the values of basic variables in Table 1 as an illustration, the size of a typical crack as a function of time can be determined from Equation (5) and shown in Fig. 2. As can be seen the crack width increases with time as expected. At the time that the concrete cylinder completely cracks, i.e., at the time to surface cracking, there is an abrupt increase in crack width, which reflects the assumed quasi-brittle nature of the concrete.

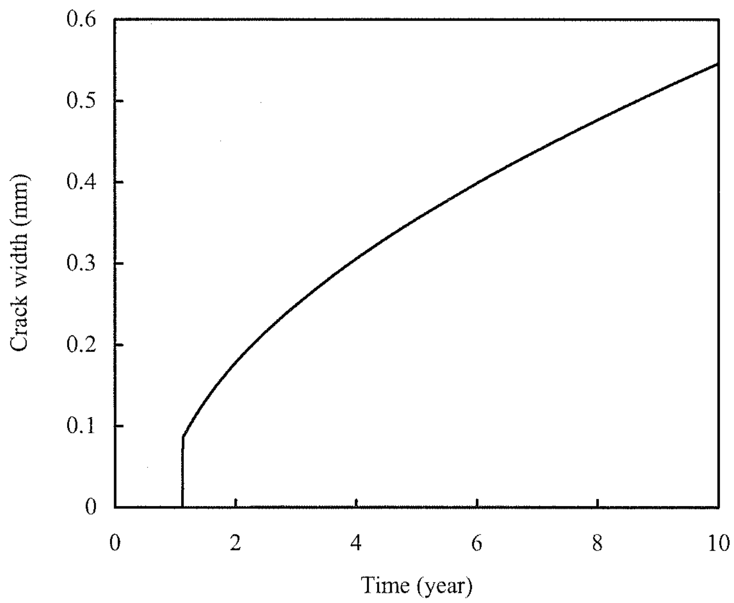

Fig. 2 Corrosion induced crack width (w_c)

Table 1 Values of basic variables used in cracking computation.

Symbol	Values
C	31 mm
D	12 mm
d_0	12.5 μm
E_c	41.62 GPa
f_t	5.725 MPa
i_{corr}	0.3686$Ln(t)$+1.1305 μA/cm^2
α_r	0.57
υ_c	0.18
ρ_r	3600 kg/m^3
ρ_{st}	7850 kg/m^3

3. Model Verification

As discussed in previous sections, most of the current research on the corrosion induced cracking process focuses on corrosion induced surface cracking (e.g., Liu and Weyers, 1998 : Pantazopoulou and Papoulia, 2001). For this reason, data on time to surface cracking were collected from the literature. To investigate the time to surface cracking of concrete structures damaged by corrosion induced internal pressure, Liu and Weyers (1998) carried out a comprehensive experiment on RC slabs subjected to chloride induced corrosion. They observed the surface cracking behavior of corrosion affected RC slabs with various concrete geometry and properties (up to five years). Their results for time to surface cracking are shown in Fig. 3. Using the same values of their test variables, the calculated time to surface cracking from the proposed model is also shown in Fig. 3. As can be seen, the analytical results are in good agreement with experimental results, with a maximum difference of about 10% for a range of different concrete covers.

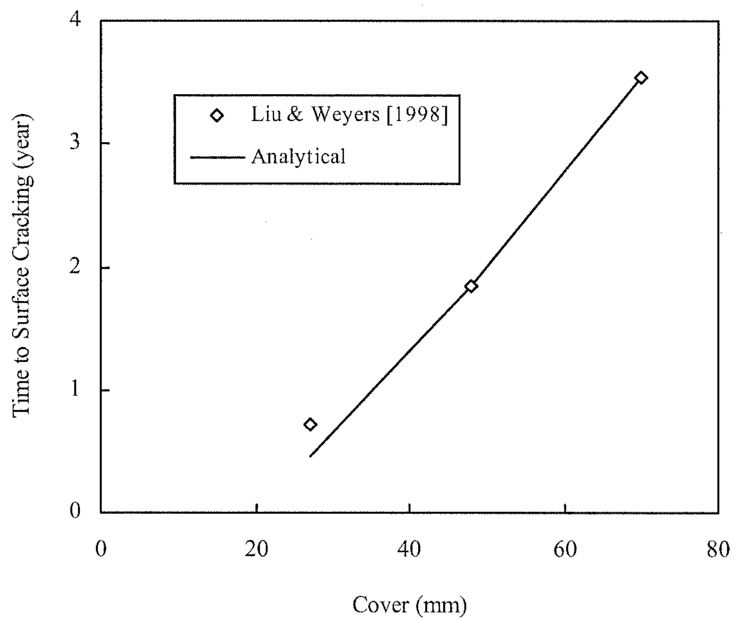

Fig. 3 Experimental verification of time to surface cracking

As noted earlier, Pantazopoulou and Papoulia (2001) developed a numerical algorithm to determine the time to surface cracking of concrete structures subjected to chloride induced corrosion. In their algorithm, the problem of corrosion induced concrete cracking is modeled as a boundary value problem and solved using a finite differences method. Also the cracks in the concrete are assumed to be smeared and the concrete is assumed to be quasi-brittle and anisotropic material. These are the same assumptions adopted in this paper and hence their results can be used for comparison. As shown in Table 2, the difference between the numerical and analytical results for crack width is about 1%.

Table 2 Comparison of time to surface cracking

Model	Time to surface cracking (in year)	Difference (in %)
Analytical	3.53	-
Experimental*	3.54	0.3
Numerical**	3.50	0.8

*Liu and Weyers (1998)
**Pantazopoulou and Papoulia (2001)

A further comparison may be made with experimental results on crack width. It may be appreciated that data on direct measurement of crack widths either from the laboratory or field are scarce. Few data are available for laboratory specimens of a practical size (Vu and Stewart, 2002). In this regard, data reported by Andrade et al. (1993) appear to be the only data useable. In their test, the specimens were 15 x 15 x 38 cm. The corrosion was accelerated by imposing electric current (as high as 100 $\mu A/cm^2$) so that the measurable crack width can be achieved within a test period of up to 100 days. Some results of their measured crack width are shown in Fig. 4. Using the same values of their test variables, e.g., corrosion rate, concrete geometry and properties, the calculated crack width is also shown in Fig. 4. As can be seen the analytical results are in reasonable agreement with experimental results. Of interest here is that almost all measured crack widths are smaller than (or equal to) the calculated crack widths, indicating that the derived model indeed gives maximum crack width as implied in Equation (5).

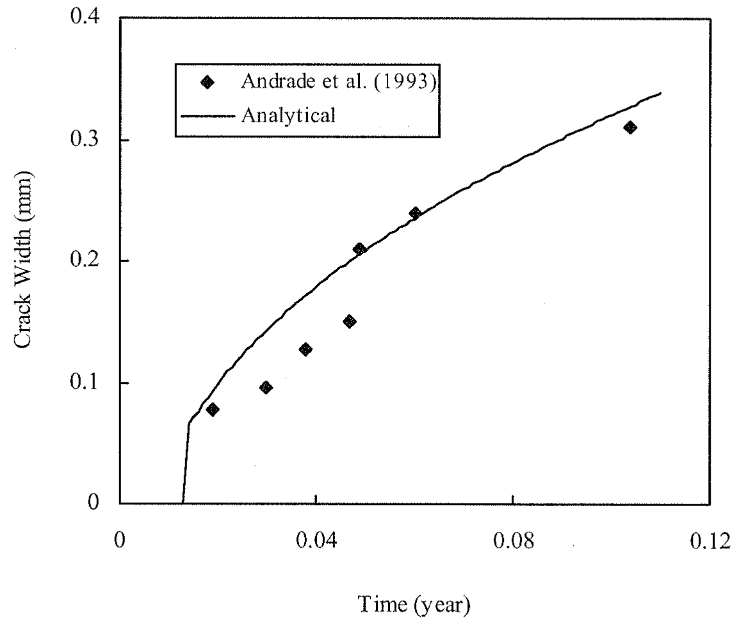

Fig. 4 Experimental verification of crack width

Conclusions

An analytical model for corrosion induced crack width in reinforced concrete structures has been proposed based on the concept of smeared cracks and verified against both experimental and numerical results. The model is directly related to critical factors that affect the corrosion induced cracking process, namely the corrosion rate, the concrete geometry and property. It can be concluded that the model presented in the paper can predict corrosion induced crack width with reasonable accuracy and therefore can serve as a useful tool for engineers, operators and asset managers in decision-making regarding the maintenance and repairs of corrosion affected reinforced concrete structures. Timely maintenance and repairs have the potential to prolong their service life.

Acknowledgements

Financial support from the Engineering and Physical Sciences Research Council (EPSRC) with Award No. GR/R28348 and the Royal Academy of Engineering with Award No. GRA 10177/93, both of UK, is gratefully acknowledged

References

1. Andrade, C., F.J. Molina and C. Alonso. 1993. Cover cracking as a function of rebar corrosion: part I – experiment test. Materials and Structures 26 : 453–464.
2. Alonso, C., C. Andrade, J. Rodriguez and J.M. Diez. 1998. Factors controlling cracking of concrete affected by reinforcement corrosion. Materials and Structures 31(8) : 435–441.
3. Bažant, Z.P. 1979. Physical model for steel corrosion in concrete sea structures – theory. Journal of Structural Division, ASCE 105(ST6) : 1137–1153.
4. Bažant, Z.P. and J. Planas. 1998. Fracture and Size Effect in Concrete and Other Quasibrittle Materials. CRC Press.
5. Bažant, Z.P. and M. Jirasek. 2002. Nonlocal integral formulations of plasticity and damage: survey and progress. Journal of Engineering Mechanics, ASCE 128(11) : 1119–1149.
6. Broomfield, J. 1997. Corrosion of Steel in Concrete, Understanding, Investigating & Repair. E & FN Spon, London.
7. Coronelli, D. 2002. Corrosion cracking and bond strength modeling for corroded bars in reinforced concrete. ACI Structural Journal 99(3) : 267–276.
8. Dagher, H.J. and S. Kulendran. 1992. Finite element modeling of corrosion damage in concrete structures. ACI Structural Journal 89(6) : 699–708.
9. Dhir, R.K. and M.J. McCarthy, (Eds). 1999. Concrete Durability and Repair Technology. Thomas Telford, London.
10. Fenner, R.T. 1989. Mechanics of Solids. Blackwell Scientific Publications, Oxford.
11. Francois, R. and G. Arliguie. 1998. Influence of service cracking on reinforcement steel corrosion. Journal of Materials in Civil Engineering, ASCE 10(1) : 14–20.
12. Kanninen, M.F. and C.H. Popelar. 1985. Advanced Fracture Mechanics. Oxford University Press, New York.
13. Lekhnitskii, S.G. 1963. Theory of Elasticity of an Anisotropic Elastic Body. Holden-Day, San Francisco.

14. Li, C.Q. 2003. Life cycle modelling of corrosion affected concrete structures –propagation. Journal of Structural Engineering, ASCE 129(6) : 753–761.
15. Li, C.Q., J.J. Zheng, W. Lawanwisut and Z.J. Chen. 2004. An analytical model for corrosion induced crack width in reinforced concrete structures. ACI Structural Journal (submitted).
16. Liu, Y. and R.E. Weyers. 1998. Modeling the time-to-corrosion cracking in chloride contaminated reinforced concrete structures. ACI Materials Journal 95(6) : 675–681.
17. Molina, F.J., C. Alonso and C. Andrade. 1993. Cover cracking as a function of rebar corrosion: part 2 – numerical model. Materials and Structures 26 : 532–548.
18. Noghabai, K. 1996. Environmental effects on bond in reinforced concrete structures. Proc. of the 7th Int. Conf. Durability of Building Materials and Components, Stockholm, Sweden 605–614.
19. Noll, W. 1972. A new mathematical theory of simple materials. Arch. Ration. Mech. Anal. 48 : 1–50.
20. Plaat, O. 1971. Ordinary Differential Equations. Holden-Day, San Francisco.
21. Pantazopoulou, S.J. and K.D. Papoulia. 2001. Modeling cover-cracking due to reinforcement corrosion in rc structures. Journal of Engineering Mechanics, ASCE 127(4) : 342–351.
22. Rodrigues, J., L.M. Ortega, J. Casal and J.M. Diez. 1996. Corrosion of reinforcement and service life of concrete structures', Proc. of the 7th Int. Conf. Durability of Building Materials and Components, Stockholm, Sweden : 117-126.
23. Sheng, J. M., Z.Z. Wang and J.J. Jiang 1991. Finite Element Methods for Reinforced Concrete and Limit Analysis of Plates and Shells. Tsinghua University Press, Beijing.
24. Tepfers, R. 1979. Cracking of concrete cover along anchored deformed reinforcing bars. Magazine of Concrete Research 31(106) : 3–12.
25. Timoshenko, S.P. and J.N. Goodier. 1970. Theory of Elasticity. McGraw-Hill Book Company, New York.
26. Thoft-Christensen, P. 2001. Corrosion crack based assessment of the life-cycle reliability of concrete structures. Proc. of Int. Cong. Structural Safety and Reliability, Corotis et al. (eds), Swets & Zeitlinger : 1–7.
27. Ueda, T., Y. Sato, Y. Kakuta and H. Kameya. 1998. Analytical study on concrete cover cracking due to reinforcement corrosion. Proceedings of an Int. Conf. Concrete Under Severe Condition 2, Tromso, Norway : 678–687.
28. Vu, K.A.T. and M.G. Stewart. 2002. Spatial variability of structural deterioration and service life prediction of reinforced concrete bridges. Proc. of Int. on Bridge Maintenance, Safety and Management - IABMAS 2002, Barcelona, CD-ROM.